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TECHNICAL NOTE

D-948

A SIMULATOR STUDY OF THE EFFECTIVENESS OF A
PILOT'S INDICATOR WHICH COMBINED ANGLE OF ATTACK
AND RATE OF CHANGE OF TOTAL PRESSURE AS APPLIED
TO THE TAKE-OFF ROTATION AND CLIMBOUT
OF A SUPERSONIC TRANSPORT

By Albert W. Hall and Jack E. Harris

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SUMMARY

A simulator study has been made to determine the effectiveness of a single instrument presentation as an aid to the pilot in controlling both rotation and climbout path in take-off. The instrument was basically an angle-of-attack indicator, biased with a total-pressure-rate input as a means of suppressing the phugoid oscillation. Linearized six-degree-of-freedom equations of motion were utilized in simulating a hypothetical supersonic transport as the test vehicle. Each of several experienced pilots performed a number of simulated take-offs, using conventional flight instruments and either an angle-of-attack instrument or the combined angle-of-attack and total-pressure-rate instrument.

The pilots were able to rotate the airplane, with satisfactory precision, to the 15° angle of attack required for lift-off when using either an angle-of-attack instrument or the instrument which combined total-pressure-rate with angle of attack. At least 4 to 6 seconds appeared to be required for rotation to prevent overshoot, particularly with the latter instrument.

The flight paths resulting from take-offs with simulated engine failures were relatively smooth and repeatable within a reasonably narrow band when the combined angle-of-attack and total-pressure-rate instrument presentation was used. Some of the flight paths resulting from take-offs with the same engine-failure conditions were very oscillatory when conventional instruments and an angle-of-attack instrument were used.

The pilots considered the combined angle-of-attack and total-pressure-rate instrument a very effective aid. Even though they could, with sufficient practice, perform satisfactory climbouts after simulated engine failure by monitoring the conventional instruments and

making corrections based on their readings, it was much easier to maintain a smooth flight path with the single combined angle-of-attack and total-pressure-rate instrument.

INTRODUCTION

The loss or deterioration of the pilot's external visual reference due to unfavorable weather or lighting conditions increases the difficulty of establishing the take-off climb. In order to avoid an oscillatory flight path which could be dangerous near the ground, the pilot must be able promptly to interpret and react to the indications of several instruments such as airspeed, altitude, rate of climb, and attitude. The attitude indicator is not reliable in the pitch direction after having been under the effect of prolonged longitudinal acceleration during the take-off run.

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Present-day turbojet transports have magnified the problem, since the higher speeds result in greater altitude changes in a given time for a given flight-path-angle change. The take-off problem could be more critical for some of the presently proposed supersonic transport configurations which require large rotation angles to develop sufficient lift. The location of the pilot far ahead of the center of gravity probably will have a serious effect on his ability to rotate quickly and precisely to the required angle by judgment based only on "feel" and "visual cues." An instrument will be required to aid the pilot in controlling the take-off rotation of a supersonic transport; in fact, instruments are currently being installed on some present-day turbojet transports for this purpose. It would be very desirable if this instrument could also be used as an aid in establishing close control of the flight path during the transition and climbout.

An angle-of-attack indication would be an aid in rotation; however, during the climbout it would not be a sufficient reference because of its inability to indicate the presence of the phugoid mode. The phugoid or long-period mode is characterized by large amplitude changes in altitude and airspeed while the angle of attack remains essentially constant.

An instrument which combines the measurement of angle of attack with an input that will allow heavy damping of the phugoid oscillation by the pilot could satisfy the requirements for both precise rotation and flight-path control. An instrument based on this principle has been developed and is described in references 1 and 2. This instrument combines angle of attack and horizontal acceleration, the latter quantity being measured by a combination of accelerometer and attitude gyro. A simple analysis indicated that a similar result could be obtained by combining angle of attack with rate of change of total pressure. The

rate of change of total pressure contains the time derivatives of velocity and altitude, both of which will provide what is known as "rate damping" for the phugoid mode. Since total-pressure rate can be measured with an inherently simple sensing device (similar to a rate-of-climb instrument), the combination of angle of attack and rate of change of total pressure was chosen for the present study.

This paper presents the results of a simulator study of an instrument based on this principle to determine its effectiveness as an aid to a pilot during the take-off rotation and climbout of a supersonic transport configuration. The particular instrument that was simulated for this study was assumed to use pressure-sensing devices to measure angle of attack as well as the rate of change of total pressure. The study was conducted with an analog computer and fixed-base-cockpit simulator utilizing linearized six-degree-of-freedom equations of motion and aerodynamic characteristics representative of one possible configuration of a supersonic transport. Several experienced test pilots made simulated take-offs with full power and with an engine failure occurring at lift-off or at a 200-foot altitude. Take-off flight profiles for the tests with a simulated engine failure are presented and comparisons are made between flights using normal instruments and an angle-of-attack indicator and flights using normal instruments and the instrument combining angle of attack and rate of change of total pressure. The comments of the pilots are presented and summarized. Atmospheric turbulence was simulated for a few take-offs in order to determine the effectiveness of the combined instrument while the angle-of-attack and total-pressure-rate signals were being influenced by turbulence.

SYMBOLS

The following symbols are used in the body of the report and in the figures. A separate symbol list is provided for the appendix.

h	altitude, ft
K	gain factor
N_Z	normalized aerodynamic force along Z body axis, positive upward, g units
\dot{p}_t	time rate of change of total pressure, where Total pressure = Static pressure + Dynamic pressure
V	true airspeed or velocity along flight path, knots
V_R	velocity at which rotation is initiated, knots

V_{35}	velocity at 35-foot altitude, knots
α	angle of attack, deg
$\alpha - K\dot{p}_t$	expression representing the instrument which combines angle of attack and rate of change of total pressure
γ	flight-path angle, deg
δ_c	canard-control-surface deflection, deg
θ	pitch attitude, deg
Subscript:	
lo	at lift-off

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DESCRIPTION OF EQUIPMENT AND TEST PROCEDURE

The study was conducted on an analog computer and fixed-base-cockpit simulator utilizing linearized six-degree-of-freedom equations of motion and aerodynamic characteristics representative of a four-engine delta-wing supersonic transport. The stability derivatives were assumed to be constant; that is, the value did not vary with angle or velocity. Ground effect on lift and induced drag was included in the simulation (based on ref. 3). The airplane was assumed to have no flaps and the lift coefficient was zero at zero angle of attack. The thrust was assumed not to vary with speed or altitude for this problem. Simulated engine failure was controlled from the computer console by a step reduction of 25 percent of the thrust. This sudden power loss produced a yawing moment and through this yaw a rolling moment was introduced, both of which required corresponding corrections to be made by the pilot, who had control in all three axes. The landing-gear retraction was simulated by means of a step reduction in the drag value and was controlled from the computer console upon the pilot's request of "gear up." No moments were introduced by this drag change.

A cockpit from a single-seat jet fighter airplane was fitted with a throttle control and spring-centered stick and rudder controls. The cockpit instrument panel is shown in figure 1. Information was presented to the pilots as indicated by the labels and scales shown in the photograph with the exception of the meter labeled Q , which was inoperative. The angle-of-attack meter was used in each run as either an angle-of-attack indicator or as an indicator for the combined angle-of-attack and total-pressure-rate terms. When used as an α indicator, the meter presented the angle of attack directly in degrees as it would be determined from a vane or other device which aligns itself with the airstream.

When the meter was used as an indicator for the combined angle and total-pressure-rate terms, the display was presented as an error indication; that is, zero represented the desired meter reading and 1 unit on the meter represented an angle of attack of about 1.2° at a speed of 200 knots. Throughout this paper the notation " α display" is used to indicate that the pilot had all of the instruments shown in figure 1 available for reference and that the angle display represented angle of attack. The notation " $(\alpha - K\dot{p}_t)$ display" is used to indicate that the angle display represented the combined angle-of-attack and total-pressure-rate terms with no change in the other instruments. A detailed discussion of the means of obtaining and combining the signals used in the $(\alpha - K\dot{p}_t)$ instrument is given in the appendix; however, it should be pointed out here that the gain factor K of the \dot{p}_t input provides a means of controlling the damping of the phugoid mode and that the value used in this study provided approximately 0.7 critical damping.

In order to simulate atmospheric turbulence in both the horizontal and vertical directions, the outputs of two Gaussian "white noise" generators were passed through low-pass filters. The filter time constants were chosen so that the frequency content of each filter output was matched to that measured for atmospheric turbulence in reference 4.

For each run the pilot's task was to initiate the take-off run, start rotation at an airspeed of 165 knots, rotate quickly to an angle of attack of 15° without overshooting more than 1° , and then climb out at an airspeed between 194 and 205 knots to an altitude of 2,000 feet. During most of these runs the pilot was faced with a thrust loss equivalent to an outboard-engine failure.

RESULTS AND DISCUSSION

Rotation

The lift coefficient available for take-off and the angle to which the airplane will have to be rotated to obtain this lift coefficient are poorly defined numbers for the supersonic transport at the present time because of the variety of configurations currently being considered. It is likely, however, that the lift coefficient available will be lower and the rotation angle higher than for present-day transports. The airplane simulated in this study represented a delta-wing configuration with an aspect ratio of 2 and a lift coefficient of 0.75 at an angle of attack of 15° . Preliminary analysis of some unpublished data has indicated that rotation angles that are as much as 1° too low can increase the take-off distance by as much as 9 percent, whereas increasing

the time required for rotation from 3 seconds to 5 seconds increased the take-off distance by only about 1 percent. Therefore, it would appear desirable to try to rotate as close as possible to the maximum angle (without dragging the tail skid) even at the expense of more time required to perform this type of rotation. During the investigation, several runs were made in which the pilot tried to achieve a very fast rotation (0° to 15° in 2 to 3 seconds). From these data it would appear that with practice a pilot could rotate the simulated airplane from an angle of attack of 0° to about 15° in 2 or 3 seconds and control the motion with no overshoot while using an angle-of-attack indication. A time history of such a rotation is presented in figure 2(a). Both the $(\alpha - K\dot{p}_t)$ signal and the α signal from the computer are shown, but the pilot's cockpit display for this run was α . The $(\alpha - K\dot{p}_t)$ signal is still some distance from the desired zero value when the angle of attack reaches its desired value. This slight lag, which is due in part to the time lag in the total-pressure-rate measurements (see appendix), was objectionable to pilots when attempting a fast rotation. Most of the time histories of fast rotations with the $(\alpha - K\dot{p}_t)$ instrument were similar to that shown in figure 2(b). The instrument lag is sufficient to cause difficulty in stopping the rotation at the desired angle without an appreciable overshoot of angle of attack. By rotating at a slower rate the pilots were able to bring the $(\alpha - K\dot{p}_t)$ indication to zero without overshooting, and the angle of attack reached the desired value without overshooting (fig. 2(c)). The $(\alpha - K\dot{p}_t)$ instrument gives an indication which should allow a pilot to make a precise well-controlled rotation from 0° to 15° in about 4 to 6 seconds. It should be mentioned here that for most of these tests in which the $(\alpha - K\dot{p}_t)$ display was used, the direction of needle deflection was such that when the needle was up the airplane nose had to be raised to bring the needle back to zero.

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Transition and Climb

The lift-off occurred when the angle of attack passed through 12.5° to 13.5° and the velocity was between 180 and 190 knots. These conditions do not give the minimum lift-off and climb velocities nor do they necessarily give the optimum conditions for the shortest take-off distance, but they are believed to represent normal operation for an airplane having the performance capabilities of the simulated airplane.

Runs made under full power conditions indicated that when the pilots became familiar with the airplane characteristics, they could consistently achieve satisfactory flight paths with the α display; however, the flight path was usually smoother and the climb velocity was stabilized with considerably less effort when the $(\alpha - K\dot{p}_t)$ display was used.

When faced with a power loss at some critical point of the take-off, the pilot's task becomes much more exacting and it is here that the benefits to be derived from the $(\alpha - K\dot{p}_t)$ display were best demonstrated.

The time histories of two successive runs made by the same pilot are presented in figures 3 and 4 and the corresponding flight paths are given in figure 5. For both runs V_R was about 165 knots, α_{10} was about 13° , V_{10} was 186 knots, V_{35} was 192 knots, and an engine failure occurred at lift-off. The $(\alpha - K\dot{p}_t)$ display was used during run 1 and the pilot was able to stabilize the velocity and flight path with very little effort in spite of the fact that the sudden power loss at lift-off produced a yaw and corresponding roll which had to be corrected. In contrast, run 2 was made with the α display. Just after lift-off and the accompanying power loss the angle of attack was too large, but before the pilot could properly correct this condition the airspeed had dropped to 185 knots ($V_{35} = 192$ knots) and the altitude was about 200 feet. In trying to increase the airspeed, 130 feet of altitude was lost and the airspeed increased to 213 knots. Close inspection of these records shows that the angle of attack was reduced as soon as it became apparent that the airspeed was beginning to drop; however, it was some time after this before the airspeed began to increase again. This time lag of airspeed with changes in angle of attack is the main reason for the difficulty in stabilizing the flight path by methods which depend primarily on observation of airspeed changes.

The flight paths of several runs during which an engine failure occurred at lift-off are presented in figure 6. The twelve runs made with the $(\alpha - K\dot{p}_t)$ display are represented by the boundaries formed by the upper and lower envelopes of the flight-path curves. The flight-path curves for seven of the twelve runs made with the α display are indicated by the dashed lines. The other five runs made with the α display were similar to runs 5 and 6; that is, they were generally within the boundary of the runs made by the $(\alpha - K\dot{p}_t)$ display and therefore are not shown here.

The flight paths of several runs during which an engine failure occurred at an altitude of 200 feet are presented in figure 7. Three of the seven runs were made with the $(\alpha - K\dot{p}_t)$ display and the other four were made with an α display. Runs 10, 14, and 15 were made by the same pilot, using the $(\alpha - K\dot{p}_t)$ display for run 10 and the α display for runs 14 and 15. During the early part of run 14 the airspeed "got away" before it was noticed and the velocity dropped to 10 knots below the lift-off velocity because the angle of attack was

not reduced for some time after the power loss. The absence of audible cues and the feel of deceleration in this simulation probably caused the simulated engine failure to remain undetected for several seconds. Run 15 was made with the pilot prepared for the engine failure and ready to monitor the necessary instruments as closely as possible to perform a steady climb, and a safe climb was made with relatively small oscillations. For this run $V_{35} = 192$ knots and the airspeed never dropped below this value, but it increased to 207 knots twice. The pilot felt that this climb was the best that could be done with the α display with his limited experience in this particular simulated airplane.

One pilot made several runs under similar engine-failure conditions with the α display until he established a schedule of altitude, attitude, and rate-of-climb check points which produced a smooth flight path. With this "flight manual" type of information fixed in his mind for a given set of conditions, he was able to climb out after an engine failure with a flight path as good as that obtained with the $(\alpha - K\dot{p}_t)$ display, though considerably more concentration and effort was required. Where conditions of weight and power may vary from flight to flight this learning process would not be available, however.

Several pilots made runs using the attitude indicator without either α or $(\alpha - K\dot{p}_t)$, and with a little practice these runs were almost as consistent as the runs made with the $(\alpha - K\dot{p}_t)$ display. It must be remembered, however, that the attitude indicator used here was presenting attitude angle as computed in the analog computer and was not affected by longitudinal acceleration. Here, as in the previous case, the practice runs gave the pilot information which was sufficient only for a particular case of weight and power conditions.

Pilots' Comments

The pilots' opinions were divided on the effectiveness of the $(\alpha - K\dot{p}_t)$ display during rotation. The lag in the instrument indication was objectionable to some pilots, particularly when the rotation was very fast and no overshoot was to be tolerated. At slower rotation rates (2° to 3° per second) the $(\alpha - K\dot{p}_t)$ display gives a satisfactory indication of the proper rotation angle.

The pilots were unanimous in their opinion that the $(\alpha - K\dot{p}_t)$ display was a very effective aid during an instrument climbout after a power failure. The pilots seemed to feel that after they had acquired sufficient familiarity with the handling qualities and performance capabilities of an airplane of this type, they could safely and consistently perform climbs, with an engine out as simulated in these

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tests, when using conventional flight instruments supplemented by an angle-of-attack indicator. However, this task would involve close monitoring of several indicators such as airspeed, altitude, rate-of-climb, and angle-of-attack indicators. The pilots felt that this task was much more difficult than the same task as performed with the single $(\alpha - K\dot{p}_t)$ instrument, particularly when such things as weight changes and the effects of temperature and altitude on power must be considered. Reference to the use of the single $(\alpha - K\dot{p}_t)$ instrument is not meant to imply that all others are ignored. The pilot is trained to scan constantly all his pertinent instruments, but when he is flying by the $(\alpha - K\dot{p}_t)$ instrument he needs to make pitch corrections through the airplane controls only as called for by the single instrument, and the other longitudinal plane instruments will then indicate the desired readings as they are scanned.

Turbulence

A few runs were made to determine the effect of turbulent air on the pressure-sensing devices employed in the $(\alpha - K\dot{p}_t)$ instrument. The simulated turbulence level had a root-mean-square velocity of 5 feet per second in both the vertical and horizontal directions. The flight-path records made during these runs through turbulent air were just as smooth as corresponding runs in smooth air. The needle oscillation equivalent to $\pm 1^\circ$ to $\pm 1.5^\circ$ on the $(\alpha - K\dot{p}_t)$ instrument presented no particular difficulty to the pilot in trying to keep the meter "zeroed," since the pilot realized he was in turbulence and did not "chase the needle" but tended to control the average needle displacement to a value near zero.

CONCLUDING REMARKS

The results of a simulator study to determine the effectiveness of a single instrument presentation, which combined angle of attack and rate of change of total pressure, as an aid to the pilot during the take-off rotation and climbout of an airplane have indicated the following for the case of the supersonic transport simulated:

1. The airplane could be rotated in 4 to 6 seconds to an angle of attack of about 15° in a precise, well-controlled manner with either an angle-of-attack instrument or the instrument which combined the rate of change of total pressure with angle of attack.

2. Each of the pilots consistently produced smooth repeatable flight paths with the airspeed nearly constant, even after engine failure, while using the instrument which combined angle of attack and rate of change of total pressure. While using conventional instruments and an angle-of-attack indicator the same pilots produced several oscillatory flight paths, and the airspeed during the climb dropped as much as 10 knots below the lift-off velocity in some cases.

3. The pilots considered the combined angle-of-attack and total-pressure-rate instrument a very effective aid. Even though they could, with sufficient practice, perform satisfactory climbouts after simulated engine failure by monitoring the conventional instruments and making corrections based on their readings, it was much easier to maintain a smooth flight path with the single combined angle-of-attack and total-pressure-rate instrument.

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4. The use of pressure-sensing devices to produce the combined signal for angle of attack and rate of change of total pressure appears to be satisfactory. Flying through moderate turbulence did not decrease the effectiveness of the instrument.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., June 30, 1961.

APPENDIX

DESCRIPTION OF PRESSURE-SENSING DEVICES USED TO MEASURE ANGLE
 OF ATTACK AND RATE OF CHANGE OF TOTAL PRESSURE, AND THE
 RELATIVE SENSITIVITIES REQUIRED IN COMBINING THESE
 MEASUREMENTS TO PROVIDE PHUGOID DAMPING

The symbols used in this appendix are defined as follows:

$$a = \frac{C_{L\alpha}}{C_{L, l_0}}$$

$$b = \left(\frac{T - D}{W} \right)_{l_0}$$

C_D drag coefficient

$C_{D\alpha}$ drag-curve slope

C_L lift coefficient

$C_{L\alpha}$ lift-curve slope

$$c = \left(\frac{C_D}{C_L} \right)_{l_0}$$

D drag; operator $\frac{d}{dt}$

$$e = \left(\frac{C_{D\alpha}}{C_L} \right)_{l_0}$$

g acceleration due to gravity

h altitude

$$i = \sqrt{-1}$$

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K	gain factor	
K_1, K_2	constants determined by the sensitivities of the pressure sensors and the display instrument	
l	length of capillary tube	
p	static pressure	
p_l	pressure measured at lower orifice in cylinder	
p_Q	pressure in volume Q	L
p_t	total pressure, $q + p$	1
\dot{p}_t	rate of change of total pressure	6
$\dot{p}_{p_t} = p_t - p_Q$		4
p_u	pressure measured at upper orifice in cylinder	4
$p_a = p_l - p_u$		
Q	a fixed volume	
q	dynamic pressure	
r	radius of capillary tube	
s	distance along flight path	
T	thrust	
t	time	
V	true airspeed or velocity along flight path	
W	weight	
α	angle of attack, unless otherwise specified	
$\Delta\alpha$	incremental change in angle of attack from some desired value	

$-\alpha_0$ angle at which angle-of-attack measuring cylinder is preset

γ flight-path angle

$$\delta = K_1 p_\alpha - K_2 \dot{p}_t$$

$$\epsilon = \frac{K_2 \tau \rho g V}{K_1 \delta q}$$

μ viscosity

ρ density of air

σ ratio of damping to critical damping

τ time constant

$$\chi = \frac{\delta}{K_1 \delta q}$$

Subscript:

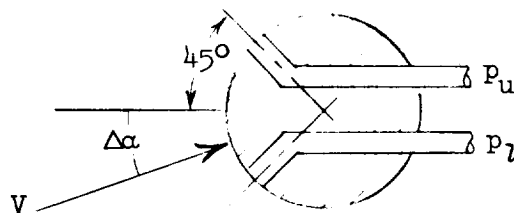
l_0 at lift-off

Superscript:

* nondimensionalized term

The proposed take-off instrument evaluated in these tests is based on pressure-sensing devices. The instrument display is driven by a signal which corresponds to angle of attack minus a given amount of rate of change of total pressure ($\alpha - K \dot{p}_t$).

The angle-of-attack measurement is assumed to come from a cylinder mounted with its axis horizontal and perpendicular to the free stream, with a pair of orifices located 90° apart on the upstream face of the cylinder, as shown in the following sketch:

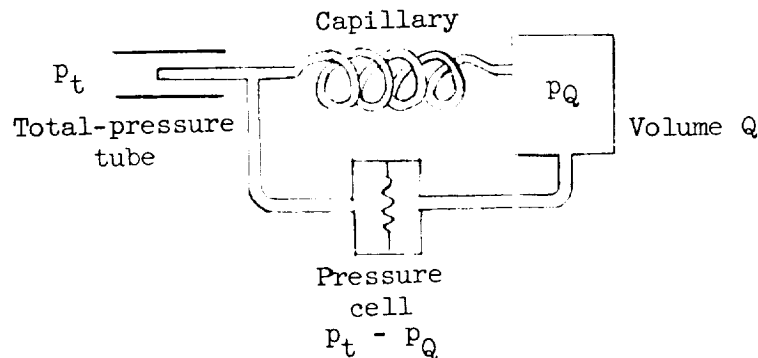


When the angle of the wind component is such that it bisects the 90° angle between the orifices, the orifice pressures p_l and p_u are equal. The angular deviation $\Delta\alpha$ of the wind component from this position is given by the following relation, which is based on the theoretical flow past a circular cylinder in a uniform stream:

$$p_l - p_u = 8q \Delta\alpha = p_\alpha$$

When the cylinder is mounted laterally from the side of a fuselage of near-circular cross section the indicated value of $\Delta\alpha$ will be about 1.5 times the actual value because of the cross flow around the fuselage and the resultant change in local flow direction. Values of p_α are independent of dynamic pressure only when this device is used as a null-seeking instrument whereby the angular position of the "nulled" cylinder is equivalent to the angle of attack. Since the purpose of this investigation is to evaluate a simple and reliable instrument, it is assumed that the cylinder is preset to some desired angle $-\alpha_0$ such that it is nulled when the airplane is rotated to the proper angle ($\Delta\alpha = \alpha + \alpha_0$).

The method of measuring total-pressure rate is similar to the method used for a rate-of-climb meter and is based on the measurement of the pressure difference across a capillary connecting the total pressure p_t and the pressure in a fixed volume p_Q , as shown in the following sketch and equation:



$$p_t - p_Q = \tau Dp_Q = \frac{\tau Dp_t}{1 + \tau} = p_{p_t} \quad (1)$$

where

$$\tau = \frac{8\mu l Q}{\pi r^4 p_t}$$

The pressure signals sensed by the two pressure-measuring devices are combined to give an instrument reading δ as follows:

$$\delta = K_1 p_\alpha - K_2 \dot{p}_t \quad (2)$$

where K_1 and K_2 are constants determined by the sensitivities of the pressure sensors and the display instrument, with the ratio K_2/K_1 establishing the amount of phugoid damping. The relation between rate of change of total pressure and airplane motion is given by the following expression:

$$\frac{dp_t}{dt} = \frac{dq}{dt} + \frac{dp}{dt} = \rho V \frac{dV}{dt} - \rho g \frac{dh}{dt} = V \frac{dq}{ds} - \rho g V \gamma$$

For convenience, the nondimensional notation

$$q^* = \frac{q - q_{l0}}{q_{l0}}$$

$$s^* = \frac{\rho g}{q_{l0}} s$$

$$D^* = \frac{d}{ds^*}$$

is used to give

$$Dp_t = \rho g V (D^* q^* - \gamma)$$

Equation (1) may be expressed as follows for low-frequency motion such as in the phugoid oscillation and for a time constant τ that is small compared with the motion frequency:

$$\dot{p}_t = \tau Dp_t$$

Equation (2) then becomes

$$\delta = K_1 \delta q \Delta \alpha - K_2 \tau \rho g V (D^* q^* - \gamma)$$

and

$$\Delta \alpha = \chi + \epsilon (D^* q^* - \gamma) \quad (3)$$

where

$$\chi = \frac{\delta}{K_1 \delta q}$$

$$\epsilon = \frac{K_2 \tau \rho g V}{K_1 \delta q}$$

The linearized perturbation equations of motion for the phugoid mode in the vertical plane only may be expressed in the following form:

$$(D^* + c)q^* + \gamma = b - e \Delta\alpha \quad (4)$$

$$-\frac{1}{2} q^* + D^* \gamma = \frac{1}{2} a \Delta\alpha \quad (5)$$

where

$$a = \frac{C_{L\alpha}}{C_{L, \gamma_0}}$$

$$b = \left(\frac{T - D}{W} \right)_{\gamma_0}$$

$$c = \left(\frac{C_D}{C_L} \right)_{\gamma_0}$$

$$e = \left(\frac{C_{D\alpha}}{C_L} \right)_{\gamma_0}$$

Eliminating q^* in equations (4) and (5) yields

$$\left[(D^*)^2 + c D^* + \frac{1}{2} \right] \gamma - \frac{1}{2} a \Delta\alpha D^* - \frac{1}{2} \left[b - \Delta\alpha(e - ac) \right] = 0 \quad (6)$$

If the airplane is flown so that an angle-of-attack indicator is held constant and equal to the angle of attack at lift-off ($\Delta\alpha = 0^\circ$), equation (6) reduces to the following differential equation:

$$\left[(D^*)^2 + c D^* + \frac{1}{2} \right] \gamma = \frac{b}{2} \quad (7)$$

whose roots are of the form

$$\lambda_{1,2} = \alpha \pm i\beta$$

where

$$\alpha = -\frac{c}{2}$$

is the damping or exponential decay term and

$$\beta = \sqrt{\frac{1}{2} - \left(\frac{c}{2}\right)^2} = \sqrt{\frac{1}{2} - \frac{1}{4}\left(\frac{C_D}{C_L}\right)_{l_0}^2}$$

is the frequency of the oscillation in terms of s^* . For the supersonic transport considered herein, $C_D/C_L = 0.178$ and $V_{l_0} = 313$ feet per second. With these values it is found that the oscillation has a period of 40 seconds and a damping ratio σ of 0.125. That is, with the use of angle of attack alone as a pilot's reference, a long-period, poorly damped, oscillatory flight path will follow take-off.

If the airplane is to be controlled by means of the instrument display represented by equation (2), the equations of motion may be written by substituting equation (3) into equations (4) and (5):

$$(D^* + c)q^* + \gamma = b - e[X + \epsilon(D^*q^* - \gamma)] \quad (8)$$

$$-\frac{1}{2}q^* + D^*\gamma = \frac{1}{2}a[X + \epsilon(D^*q^* - \gamma)] \quad (9)$$

The airplane is to be controlled so that the instrument display δ remains constant. The values of X and ϵ will be nearly constant and for this simplified analysis are assumed to be constant. Then the simultaneous solution of equations (8) and (9) gives

$$\left[(D^*)^2 + \frac{a\epsilon + c}{1 + e\epsilon} D^* + \frac{1}{2} \frac{1 + \epsilon(ac - e)}{1 + e\epsilon} \right] \gamma = \frac{1}{2} \frac{b + X(ac - e)}{1 + e\epsilon}$$

which is of the same form as equation (7). The damping term was previously equal to $-\frac{c}{2}$ or $\frac{1}{2}\left(\frac{C_D}{C_L}\right)_{l_0}$; it now has the value

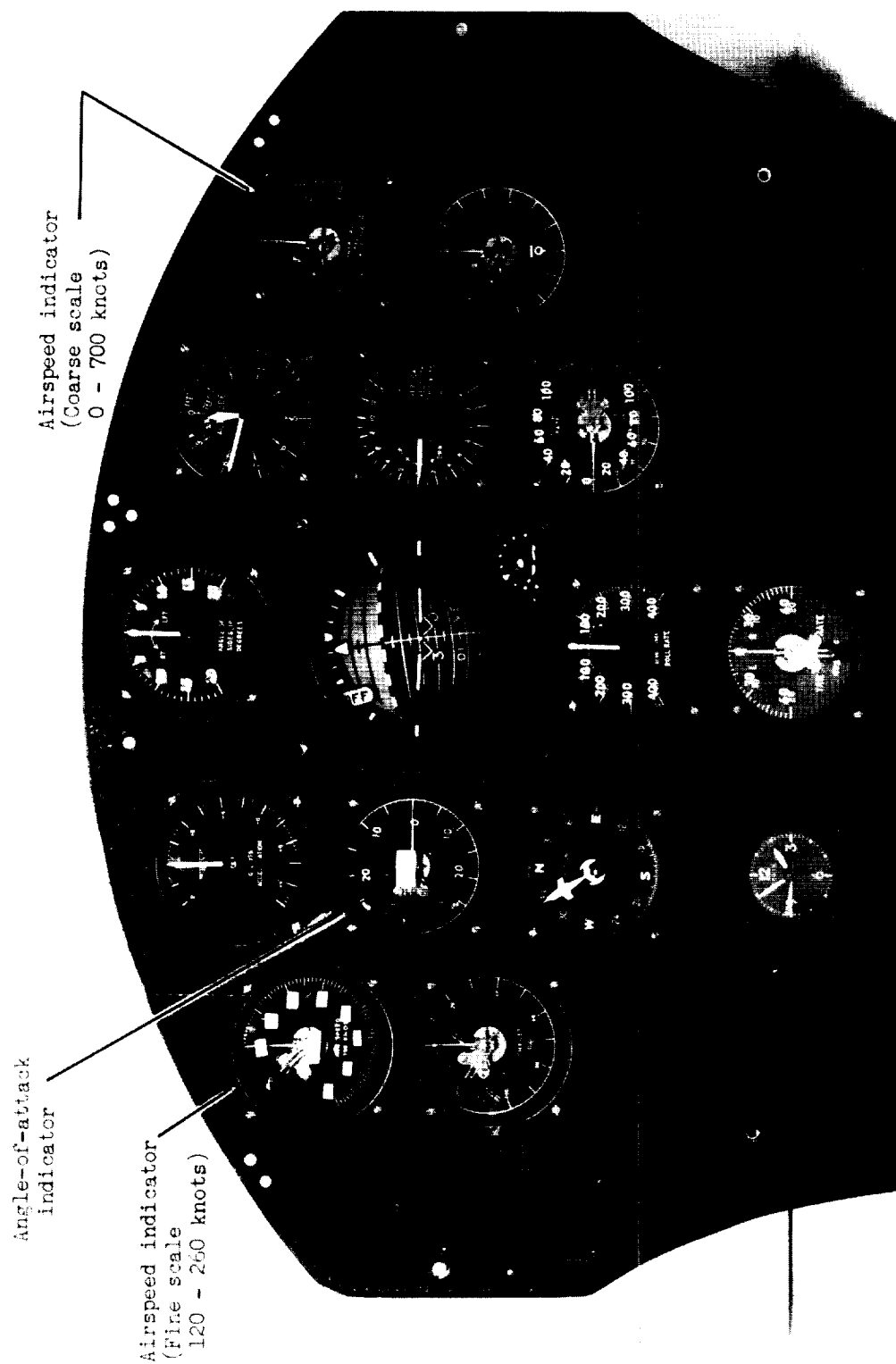
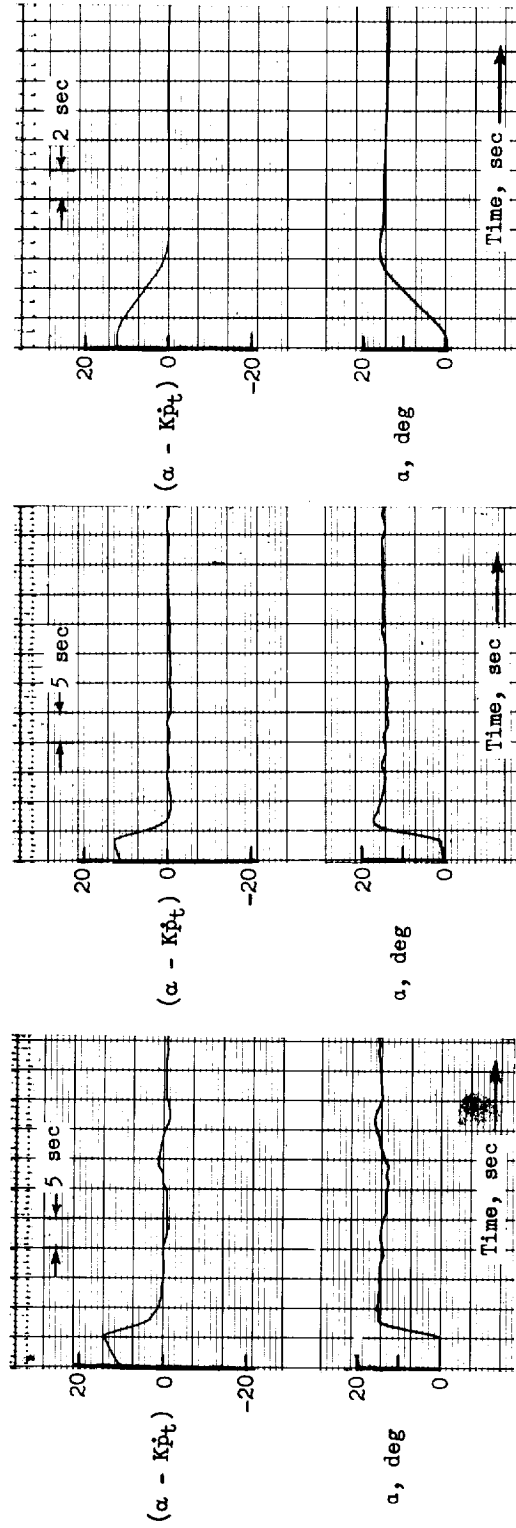
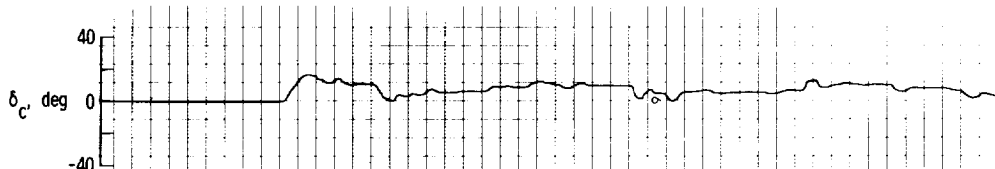


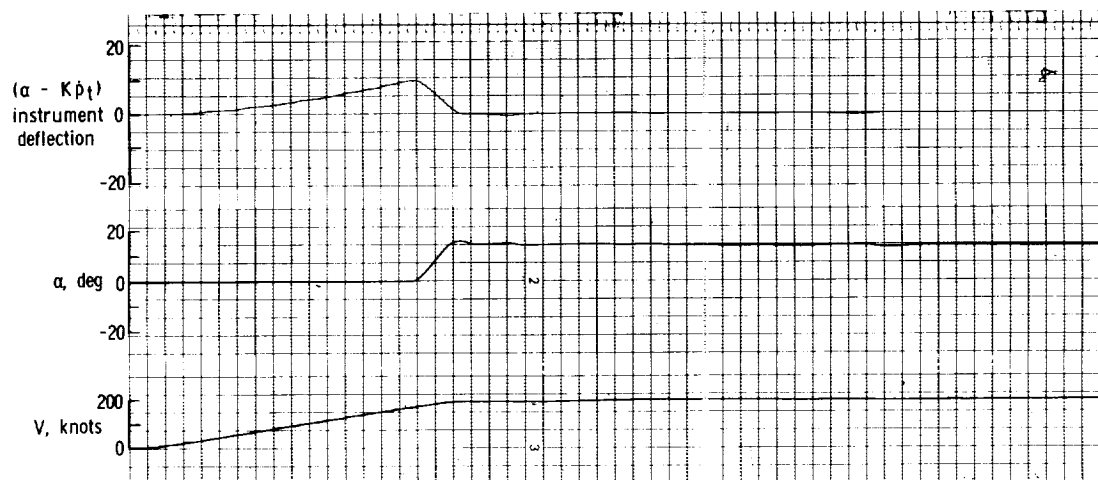
Figure 1.- Simulator instrument panel.

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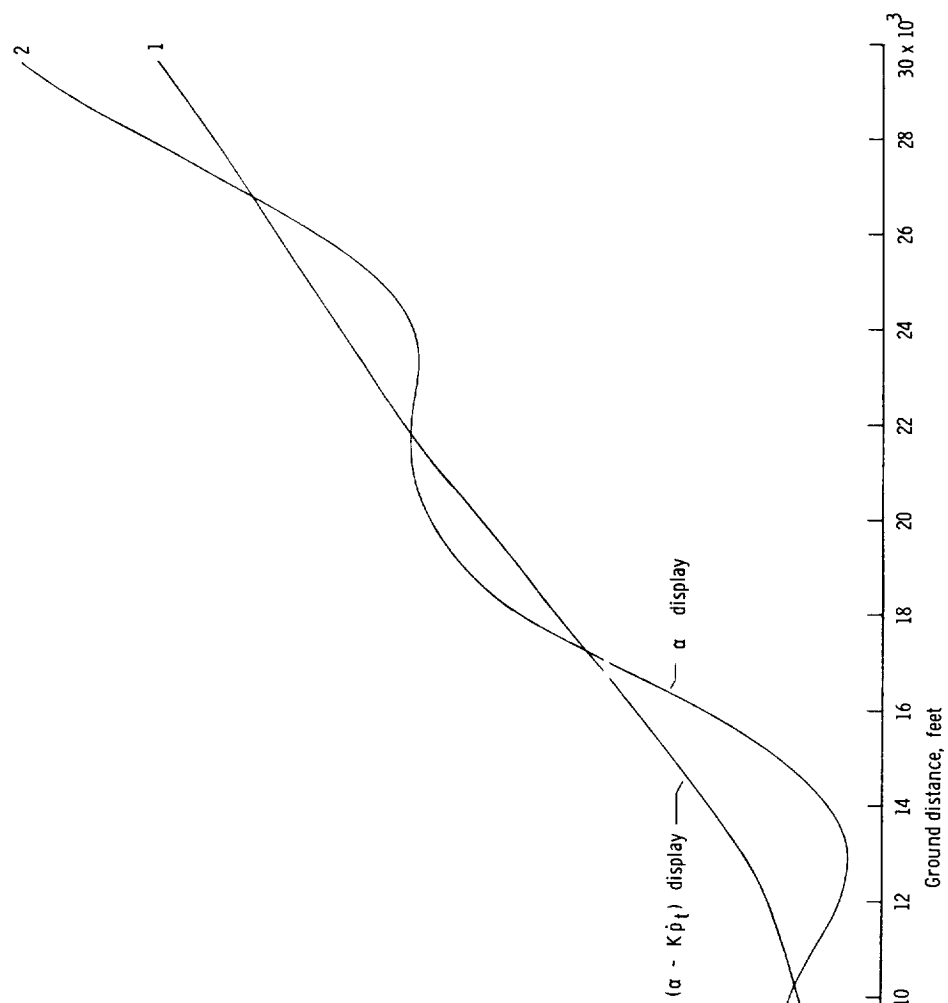


(a) α display (fast rotation).
 (b) $(\alpha - K\dot{\alpha}_t)$ display (fast rotation).
 (c) $(\alpha - K\dot{\alpha}_t)$ display (slow rotation).

Figure 2.- Time histories of α signal and $(\alpha - K\dot{\alpha}_t)$ signal. Note the time-scale change in part (c).



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t paths. Runs 1 and 2; engine failure occurring at lift-off.

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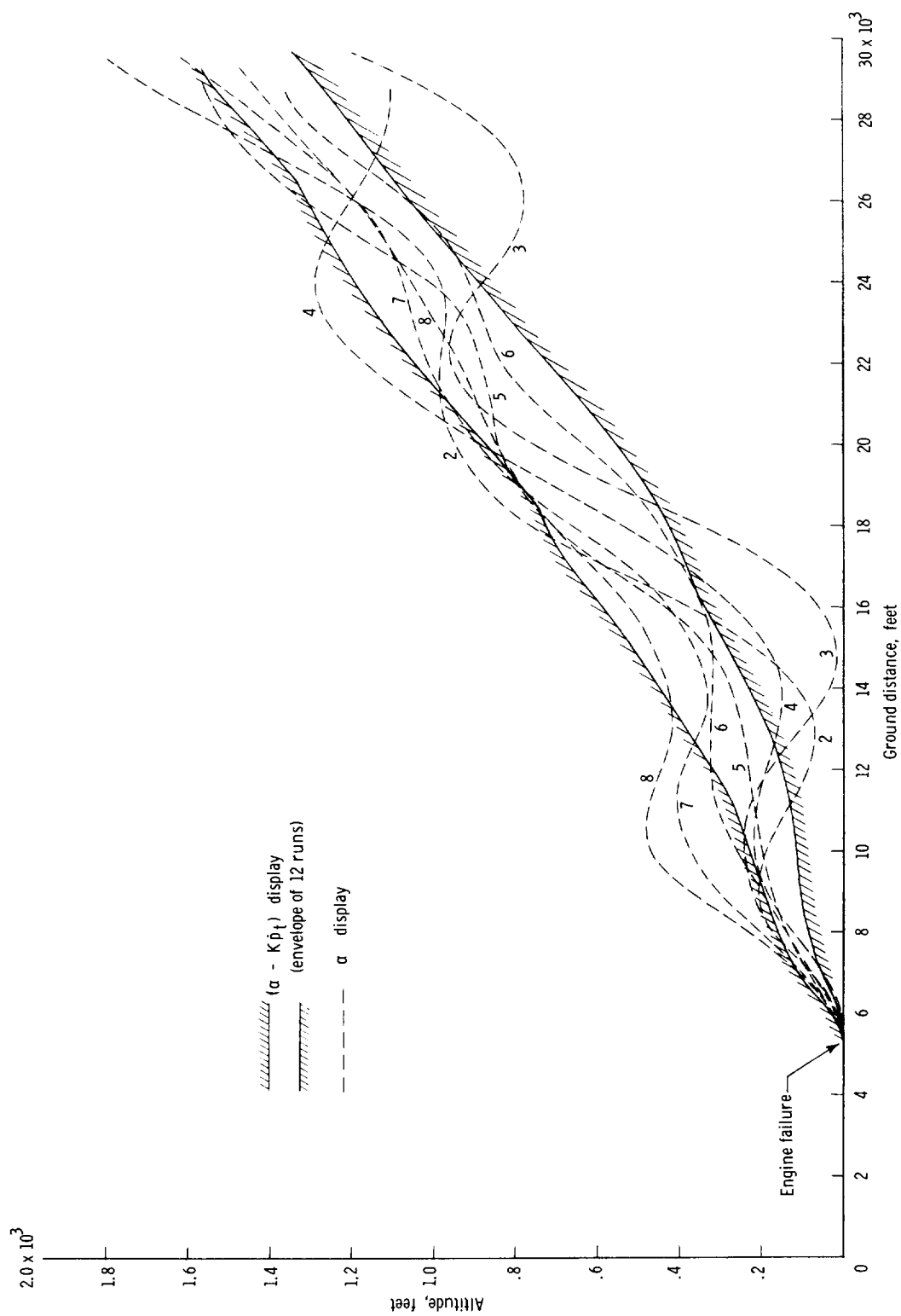


Figure 6.- Take-off flight paths in which the pilots used either the α display or the $(\alpha - K\dot{p}_t)$ display. Engine failure occurring at lift-off.

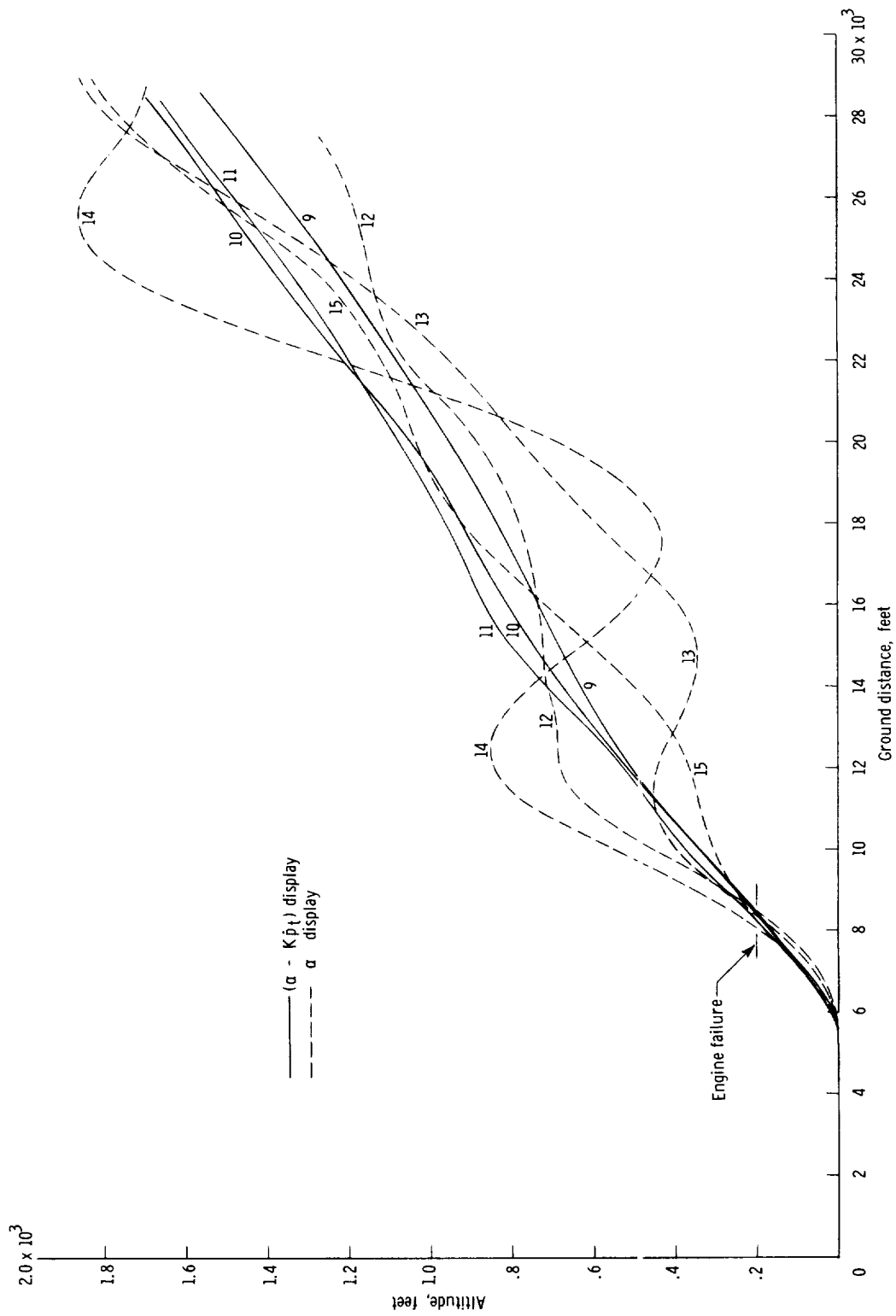


Figure 7.- Take-off flight paths in which the pilots used either the α display or the $(\alpha - K \dot{p}_t)$ display. Engine failure occurring at 200-foot altitude.